

## Portfolio analysis (Risk, Return, Sharpe, Beta)

**FINANCE TOYS**

This is a step-by-step description of some aspects of a portfolio analysis

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We will create a portfolio of several stocks and try to apply some analysis approaches to it. But that's not all. We will use 2009 weekly data for the portfolio analysis. After that we will change the portfolio depending on the results of analysis to optimize it and then check if our changes helped to improve the performance vs original portfolio and vs S&P index in 2010.

**Note:** we will analyze weekly data, which implies that we are planning to create a portfolio for one week. If you want to create a portfolio for a month, quarter, year - you need to use monthly, quarterly and annual data, respectively. Usually investors create portfolios for at least several months. We just use weekly data for convenience (2008 market distorts all the statistics and analysis)

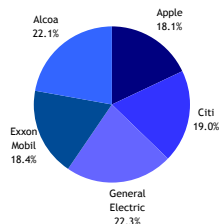
### STEP 1 CREATING A PORTFOLIO

Let's create a simple portfolio consisting of 5 stocks (the number of stocks as well as the stocks themselves are just taken from the head). We take Apple (AAPL), Citi (C), General Electric (GE), Exxon Mobil (XOM) and Alcoa (AA). I tried to pick the stocks from different sectors.

Let's imagine that currently we are at the end of 2009. We will analyze 2009 data, modify our portfolio and see what will happen to it in 2010.

Here it is, our portfolio:

Company	Ticker	Last price (end of 2009), \$	Number of shares	Position \$	Share in portfolio
Apple	AAPL	209.04	12	2 508	18.1%
Citi	C	3.35	785	2 630	19.0%
General Electric	GE	15.44	200	3 088	22.3%
Exxon Mobil	XOM	68.66	37	2 540	18.4%
Alcoa	AA	16.34	187	3 056	22.1%
<b>Total</b>				<b>13 822</b>	<b>100.0%</b>



### STEP 2 CALCULATING PORTFOLIO PARAMETERS

Let's look at the expected return and risk for our portfolio

We uploaded weekly prices for our stocks and S&P index for 2009 (to the right) and calculated weekly changes for all instruments and for portfolio (we will need this row later)

Expected return is an average weekly return

Risk is standard deviation of weekly returns (use STDEV function in excel)

So here is the stocks' parameters and S&P index parameters

	Share in portfolio	Expected return (Re)	Risk (σ)
Apple	18.1%	1.8%	4.9%
Citi	19.0%	0.4%	19.1%
General Electric	22.3%	0.3%	9.0%
Exxon Mobil	18.4%	-0.2%	3.3%
Alcoa	22.1%	1.2%	8.6%
<b>S&amp;P</b>		<b>0.6%</b>	<b>3.6%</b>

Now we will perform operations similar to what we did in Markowitz portfolio calculation

<http://www.financetoy.com/portfolio/markeng.htm>

### STEP 3 CALCULATING THE RISK AND RETURN FOR PORTFOLIO

The primal aim of portfolio creation is the diversification of non-systematic risks

Let's find the average risk and return of the portfolio and compare it with the respective parameters of S&P index

The formula for the average portfolio return is  
 $R_p = \sum (R_e \times w_s)$

i.e. you multiply all the stocks' expected returns on its weights in portfolio and sum up them

$R_p = 0.7\%$

The expected weekly return of the portfolio is higher than S&P index's expected return (0.6%)

This is good. Potentially our portfolio can outperform index on a weekly basis

Let's calculate the risk

Here is the formula for portfolio risk calculation:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{i,j}$$

Where  $\sigma$  is a risk of a stock (calculated as it's standard deviation)

$w$  - is a share of the stock in the portfolio

$\rho$  is a correlation of the stocks in the portfolio

After you find this, don't forget to get a square root from it

Let's calculate the risk in several steps

	Prices (\$)						Change (%)						Portfolio
	AAPL	C	GE	XOM	AA	S&P	AAPL	C	GE	XOM	AA	S&P	
09.01.2009	90.6	6.8	16.0	77.6	10.8	881.7							
16.01.2009	82.3	3.5	14.0	78.1	9.4	843.7	-9.1%	-48.1%	-12.8%	0.7%	-12.8%	-4.3%	-22.8%
23.01.2009	88.4	3.5	12.0	78.0	8.3	834.0	7.3%	-0.9%	-13.8%	-0.1%	-11.7%	-1.2%	-4.9%
30.01.2009	90.1	3.6	12.1	76.5	7.8	816.5	2.0%	2.3%	0.8%	-2.0%	-6.5%	-2.1%	-0.5%
06.02.2009	99.7	3.9	11.1	80.3	8.4	868.6	10.6%	10.1%	-8.5%	5.0%	7.8%	6.4%	4.2%
13.02.2009	99.2	3.5	11.4	74.6	7.5	815.3	-0.6%	-10.7%	3.1%	-7.2%	-11.0%	-6.1%	-5.9%
20.02.2009	91.2	2.0	9.4	71.2	6.3	776.6	-8.0%	-44.1%	-18.0%	-4.5%	-15.9%	-4.7%	-19.9%
27.02.2009	89.3	1.5	8.5	67.9	6.2	725.6	-2.1%	-23.1%	-9.3%	-4.7%	-1.0%	-6.6%	-8.2%
06.03.2009	85.3	1.0	7.1	64.0	5.2	679.4	-4.5%	-31.3%	-17.0%	-5.7%	-16.2%	-6.4%	-13.6%
13.03.2009	95.9	1.8	9.6	67.2	5.7	762.9	12.5%	72.8%	36.3%	5.0%	9.8%	12.3%	21.9%
20.03.2009	101.6	2.6	9.5	66.1	6.5	783.4	5.9%	47.2%	-0.8%	-1.7%	14.1%	2.7%	10.2%
27.03.2009	106.9	2.6	10.8	70.0	7.8	805.6	5.2%	0.0%	13.0%	5.9%	19.3%	2.8%	7.8%
03.04.2009	116.0	2.9	10.9	70.4	8.2	837.6	8.6%	8.8%	1.5%	0.7%	4.7%	4.0%	4.3%
10.04.2009	119.6	3.0	11.3	69.8	8.9	854.0	3.1%	6.7%	3.6%	-0.9%	8.3%	2.0%	3.8%
17.04.2009	123.4	3.7	12.4	66.8	9.3	868.3	3.2%	20.1%	9.4%	-4.4%	4.6%	1.7%	6.8%
24.04.2009	123.9	3.2	12.1	66.6	9.1	857.9	0.4%	-12.6%	-2.3%	-0.3%	-1.3%	-1.2%	-4.0%
01.05.2009	127.2	3.0	12.7	68.0	9.7	880.4	2.7%	-6.9%	-4.8%	2.2%	6.0%	2.6%	1.3%
08.05.2009	129.2	4.0	14.5	70.8	10.0	923.0	1.5%	35.4%	14.5%	4.1%	3.3%	4.8%	12.9%
15.05.2009	122.4	3.5	12.9	69.1	9.0	893.2	-5.2%	-13.4%	-11.5%	-2.4%	-9.8%	-3.2%	-9.0%
22.05.2009	122.5	3.7	13.1	68.8	8.9	881.6	0.1%	5.5%	1.9%	-0.4%	-1.9%	-1.3%	1.4%
29.05.2009	135.8	3.7	13.5	69.4	9.2	927.8	10.9%	1.4%	2.9%	0.8%	4.1%	5.2%	3.2%
05.06.2009	144.7	3.5	13.5	73.0	10.9	935.0	6.5%	-7.0%	0.4%	5.2%	18.7%	0.8%	3.2%
12.06.2009	137.0	3.5	13.5	73.8	12.0	935.2	-5.3%	0.3%	-0.2%	1.1%	9.6%	0.0%	1.1%
19.06.2009	139.5	3.2	12.1	71.1	11.0	912.6	1.8%	-8.6%	-10.4%	-3.7%	-8.3%	-2.4%	-6.4%
26.06.2009	142.4	3.0	11.8	69.1	10.8	920.9	2.1%	-4.4%	-2.9%	-2.8%	-2.3%	0.9%	-2.3%
03.07.2009	140.0	2.9	11.5	68.5	9.9	890.4	-1.7%	-5.0%	-2.5%	-0.8%	-8.4%	-3.3%	-3.6%
10.07.2009	138.5	2.6	10.8	65.1	9.3	881.9	-1.1%	-10.1%	-5.9%	-4.9%	-5.3%	-1.0%	-5.7%
17.07.2009	151.8	3.0	11.7	68.5	10.2	945.4	9.6%	16.6%	8.1%	5.2%	9.4%	7.2%	9.6%
24.07.2009	160.0	2.7	12.0	72.3	11.0	977.6	5.4%	-9.6%	3.3%	5.5%	7.8%	3.4%	2.2%
31.07.2009	163.4	3.2	13.4	70.4	11.8	996.5	2.1%	16.1%	11.4%	-2.6%	6.7%	1.9%	6.5%
07.08.2009	165.5	3.9	14.7	69.5	13.0	1 005.0	1.3%	21.5%	9.7%	-1.3%	10.5%	0.9%	8.5%
14.08.2009	166.8	4.0	13.9	68.2	13.3	986.8	0.8%	4.9%	-5.3%	-1.8%	2.1%	-1.8%	0.1%
21.08.2009	169.2	4.7	14.2	69.9	12.6	1 028.6	1.5%	16.3%	2.1%	2.5%	-5.4%	4.2%	4.1%
28.08.2009	170.1	5.2	14.1	70.1	12.5	1 020.3	0.5%	11.3%	-0.9%	0.3%	-0.5%	-0.8%	2.9%
04.09.2009	170.3	4.9	13.9	69.2	12.2	1 025.7	0.2%	-7.3%	-1.5%	-1.3%	-2.6%	0.5%	-3.1%
11.09.2009	172.2	4.6	14.7	70.0	13.0	1 036.4	1.1%	-4.9%	5.8%	1.2%	6.7%	1.0%	1.3%
18.09.2009	185.0	4.3	16.5	70.0	14.1	1 067.1	7.5%	-7.6%	12.5%	0.0%	8.2%	3.0%	3.3%
25.09.2009	182.4	4.4	16.4	68.7	13.1	1 045.4	-1.4%	2.8%	-0.8%	-1.8%	-7.0%	-2.0%	-1.4%
02.10.2009	184.9	4.5	15.4	66.6	12.8	1 027.9	1.4%	3.2%	-6.2%	-3.1%	-2.0%	-1.7%	-1.4%
09.10.2009	190.5	4.6	16.2	69.3	14.2	1 075.1	3.0%	2.4%	5.3%	4.0%	11.1%	4.6%	5.0%
16.10.2009	188.1	4.6	16.1	73.1	14.0	1 088.6	-1.3%	-0.9%	-0.6%	5.6%	-1.4%	1.3%	0.2%
23.10.2009	203.9	4.5	15.2	73.6	13.7	1 082.9	8.4%	-2.8%	-5.5%	0.6%	-2.2%	-0.5%	-0.9%
30.10.2009	188.5	4.1	14.3	71.7	12.4	1 042.0	-7.6%	-8.3%	-6.2%	-2.6%	-9.5%	-3.8%	-6.9%
06.11.2009	194.3	4.1	15.3	72.6	12.9	1 077.0	3.1%	-0.7%	7.5%	1.3%	3.8%	3.4%	2.9%
13.11.2009	204.5	4.1	15.7	72.5	13.2	1 100.8	5.2%	-0.2%	2.2%	-0.2%	2.2%	1.7%	1.7%
20.11.2009	199.9	4.2	15.6	74.4	13.1	1 094.9	-2.2%	3.7%	-0.4%	2.6%	-0.4%	-0.5%	0.8%
27.11.2009	200.6	4.1	15.9	74.9	12.7	1 091.1	0.3%	-3.3%	2.2%	0.7%	-3.6%	-0.3%	-0.7%
04.12.2009	193.3	4.1	16.2	74.3	13.0	1 105.0	-3.6%	0.0%	1.6%	-0.8%	2.6%	1.3%	0.0%
11.12.2009	194.7	4.0	15.9	72.8	14.6	1 111.1	0.7%	-2.7%	-1.7%	-1.9%	12.5%	0.6%	0.9%
18.12.2009	195.4	3.4	15.6	68.2	14.6	1 108.2	0.4%	-13.9%	-2.1%	-6.3%	-0.2%	-0.3%	-4.7%
25.12.2009	209.0	3.4	15.4	68.7	16.3	1 127.6	7.0%	-1.5%	-1.0%	0.7%	12.1%	1.7%	3.3%

First, we need to calculate the correlations between the stocks

Use CORREL function in excel to do this

**Correlation matrix**

	AAPL	C	GE	XOM	AA
AAPL	1.000	0.574	0.501	0.463	0.587
C	0.574	1.000	0.729	0.357	0.517
GE	0.501	0.729	1.000	0.449	0.619
XOM	0.463	0.357	0.449	1.000	0.550
AA	0.587	0.517	0.619	0.550	1.000

Now let's draw a couple more matrices to simplify valuation. We do all this just following the formula for portfolio risk calculation shown above

**Share matrix (matrix with market share of the stocks in portfolio)**

	AAPL	C	GE	XOM	AA
18.1%	19.0%	22.3%	18.4%	22.1%	
18.1%	19.0%	22.3%	18.4%	22.1%	
18.1%	19.0%	22.3%	18.4%	22.1%	
18.1%	19.0%	22.3%	18.4%	22.1%	
18.1%	19.0%	22.3%	18.4%	22.1%	

**Weights multiplication matrix (to calculate  $w_i \times w_j$  from the formula for portfolio risk)**

	AAPL	C	GE	XOM	AA
AAPL	0.033	0.035	0.041	0.033	0.040
C	0.035	0.036	0.043	0.035	0.042
GE	0.041	0.043	0.050	0.041	0.049
XOM	0.033	0.035	0.041	0.034	0.041
AA	0.033	0.035	0.041	0.034	0.041

**Risk matrix**

	AAPL	C	GE	XOM	AA
4.9%	19.1%	9.0%	3.3%	8.6%	
4.9%	19.1%	9.0%	3.3%	8.6%	
4.9%	19.1%	9.0%	3.3%	8.6%	
4.9%	19.1%	9.0%	3.3%	8.6%	
4.9%	19.1%	9.0%	3.3%	8.6%	

**Risk multiplication matrix (to calculate  $\sigma_i \times \sigma_j$  from the formula for portfolio risk)**

	AAPL	C	GE	XOM	AA
AAPL	0.002	0.009	0.004	0.002	0.004
C	0.009	0.037	0.017	0.006	0.016
GE	0.004	0.017	0.008	0.003	0.008
XOM	0.002	0.006	0.003	0.001	0.003
AA	0.004	0.016	0.008	0.003	0.007

Now we multiply three matrices' values: correlation matrix, weights multiplication matrix and risk multiplication matrix

**Final multiplication matrix**

	AAPL	C	GE	XOM	AA
AAPL	0.00008	0.00018	0.00009	0.00002	0.00010
C	0.00018	0.00133	0.00054	0.00008	0.00036
GE	0.00009	0.00054	0.00041	0.00006	0.00024
XOM	0.00002	0.00008	0.00006	0.00004	0.00006
AA	0.00008	0.00030	0.00020	0.00005	0.00030

I made up the matrices' names by myself. Actually, they don't have official names

Now we simply need to sum up all the values from the final matrix to get  $\sigma^2$

$\sigma^2 = 0.00547$

To get a risk for our portfolio we simply need to get a square root from this figure

$\sigma = 7.4\%$

Now we see that the risk of our portfolio is higher than for S&P index (3.6%)

But at the same time our portfolio offer better expected return. What should we do in that situation?

Just go to the next step for an answer

**STEP 4 SHARPE RATIO (SR)**

Sharpe ratio (SR) is a measure of excess return over a unit of risk

To learn more about Sharpe ratio read this Wikipedia article. It's good [http://en.wikipedia.org/wiki/Sharpe\\_ratio](http://en.wikipedia.org/wiki/Sharpe_ratio)

We will calculate Sharpe ratio using the formula

$SR = (Re - Rf) / \sigma$

where Re is the expected asset's return

Rf is a risk-free rate

and  $\sigma$  is a standard deviation, or risk of the asset

As a risk free rate we take 10Y US treasuries yield at the end of 2009 and divide it by the number of weeks in a year to get a weekly risk-free return

$Rf = 3.807\% / 52 = 0.1\%$

Let's calculate this coefficient for all the stocks in the portfolio, for the portfolio itself and for S&P index. We already know all the inputs, so it should be easy

	Expected return (Re)	Risk ( $\sigma$ )	Risk-free rate (Rf)	Sharpe ratio (SR)
Apple	1.8%	4.9%	0.1%	0.35
Citi	0.4%	19.1%	0.1%	0.02
General Electric	0.3%	9.0%	0.1%	0.03
Exxon Mobil	-0.2%	3.3%	0.1%	-0.08
Alcoa	1.2%	8.6%	0.1%	0.13
Portfolio	0.7%	7.4%	0.1%	0.08
S&P index	0.6%	3.6%	0.1%	0.13

Investors should choose the assets with the highest Sharpe ratio as it assumes higher excess return for a unit of risk

In our case Apple would be the best choice

**STEP 5 BETA ( $\beta$ )**

Beta describes the relation of the asset's return to the market return

If Beta = 1, the price for the asset changes in the same way as the market index.

If Beta = 0, there are no relation between the asset and the market

If Beta = -1, the asset's and the market's price go in opposite directions

If Beta > 1, the price of the asset changes more than by 1% for every market movement by 1%

If Beta > 1, the price of the asset falls by more than 1% every time market goes up by 1%; and it growth more than by 1% every time the market falls by 1%

Want to know about Beta? Go here [http://en.wikipedia.org/wiki/Beta\\_coefficient](http://en.wikipedia.org/wiki/Beta_coefficient)

Beta equals the covariance between the asset's and market's returns divided by market's variance

$$\beta_a = \frac{\text{Cov}(r_a, r_p)}{\text{Var}(r_p)}$$

Ra - return of asset

Rp - return of the market (or of the portfolio)

For covariance use COVAR function in excel and for variance - VAR function

Below is the calculation of Beta for each stock and for portfolio

	Covariance	Variance	Beta
Apple	0.00128	0.00130	0.99
Citi	0.00484	0.00130	3.71
General Electric	0.00235	0.00130	1.80
Exxon Mobil	0.00081	0.00130	0.62
Alcoa	0.00207	0.00130	1.59
Portfolio	0.00219	0.00130	1.68

You might notice that Citi has a Beta equal to 3.71. That's unusual. But if you remember 2009 was the crisis year and banking sector demonstrated the highest volatility

## STEP 6 DECISION MAKING

Let's insert all the parameters we found in a single table

	Share in portfolio	Expected return (Re)	Risk (σ)	Sharpe ratio	Beta
Apple	18.1%	1.8%	4.9%	0.35	0.99
Citi	19.0%	0.4%	19.1%	0.02	3.71
General Electric	22.3%	0.3%	9.0%	0.03	1.80
Exxon Mobil	18.4%	-0.2%	3.3%	-0.08	0.62
Alcoa	22.1%	1.2%	8.6%	0.13	1.59
Portfolio		0.7%	7.4%	0.08	1.68
S&P index		0.6%	3.6%	0.13	

The first thing you might notice is that the Sharpe ratio for our portfolio is lower than for S&P index which means that the latter provide better relation of risk and return.

At the same time our portfolio has Beta equal to 1.68 which means that if the market grows, our portfolio grows faster

Let's make the following changes:

decrease the share of a stock with the lowest Sharpe ratio (Exxon Mobil)

decrease the share of a stock with the lowest Beta (for example, it is because we expect S&P to grow and want to have more stocks which will outpace the index) (also Exxon Mobil)

increase the share of a stock with a highest Beta (Citi)

increase the share of a stock with a highest Sharpe ratio (Apple)

We keep Alcoa's and General Electric weights the same

## STEP 7 PORTFOLIO TESTING

Now let's test our portfolios on 2010 data

Here is our old portfolio.

	Ticker	Last price (end of 2009), \$	Number of shares	Position \$	Share in portfolio
Apple	AAPL	209.04	12	2 508	18.1%
Citi	C	3.35	785	2 630	19.0%
General Electric	GE	15.44	200	3 088	22.3%
Exxon Mobil	XOM	68.66	37	2 540	18.4%
Alcoa	AA	16.34	187	3 056	22.1%
Portfolio				13 822	100.0%

And here is a new portfolio with all the suggested changes (look at the number of shares)

	Ticker	Last price (end of 2009), \$	Number of shares	Position \$	Share in portfolio
Apple	AAPL	209.04	17	3 554	25.7%
Citi	C	3.35	1000	3 350	24.3%
General Electric	GE	15.44	200	3 088	22.4%
Exxon Mobil	XOM	68.66	11	755	5.5%
Alcoa	AA	16.34	187	3 056	22.1%
Portfolio				13 803	100%

We changed the number of shares for Apple, Citi and Exxon Mobil to get approximately the same total position in \$

After that let's calculate weekly returns for old and new portfolios (to the right)

Prices (\$)						Change (%)							
AAPL	C	GE	XOM	AA	S&P	AAPL	C	GE	XOM	AA	S&P	Old portfolio	New portfolio

