Huang-Litzenberger approach allows us to find mathematically efficient set of portfolios.

**Assumptions**

- There are no limitations on the positions' volumes.
- You can have both long (buy stocks) and short (sell stocks) positions in your portfolio.
- We use weekly data, so the expected return is a weekly return and risk is a weekly risk.

**Step 1. Download stocks’ pricing data**

First, we download pricing data for the stocks.

For my model, I've chosen four stocks from different sectors (to create a diversified portfolio):

- **APPLE**
- **CITI GROUP**
- **GENERAL ELECTRIC**
- **EXXON MOBIL**

**Step 2. Find expected return and standard deviation (risk) for each stock**

Below see the table with basic inputs for the model.

<table>
<thead>
<tr>
<th>Stock</th>
<th>e</th>
<th>d</th>
<th>w</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLE</td>
<td>1.1%</td>
<td>4.9%</td>
<td>114.34%</td>
</tr>
<tr>
<td>CITI GROUP</td>
<td>-0.1%</td>
<td>4.6%</td>
<td>-9.76%</td>
</tr>
<tr>
<td>GENERAL ELECTRIC</td>
<td>0.33%</td>
<td>4.5%</td>
<td>-124.80%</td>
</tr>
<tr>
<td>EXXON MOBIL</td>
<td>0.05%</td>
<td>2.8%</td>
<td>120.21%</td>
</tr>
</tbody>
</table>

**Step 3. Draw two unit-vectors**

We need them for interim calculations.

First: number of columns = 1; number of rows = 4 (same as the number of stocks).

Second: number of columns = 4; number of rows = 1.

All the values in vectors equal to 1.

**Unity vector**

<table>
<thead>
<tr>
<th>e</th>
<th>uT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Step 4. Draw two transposed matrices for expected returns and weights**

In table 1 (Step 2) with basic inputs you can see two columns: e (expected return) and w (weights).

Transpose simply means that you should turn columns into rows.

Make links from this matrix to Table 1 values.

We add “T” latter in the names of transposed matrices, thus we get wT and eT.
Step 5. Create covariance matrix

Covariation defines the dependence of one stock from the other. In a covariance matrix we calculate covariation between all stocks. We use COVAR excel function (details on that function and on covariation are available in Excel help). We call this covariance matrix $V$.

<table>
<thead>
<tr>
<th></th>
<th>APPLE</th>
<th>CITI GROUP</th>
<th>GENERAL ELECTRIC</th>
<th>EXXON MOBIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLE</td>
<td>0.00234</td>
<td>0.00194</td>
<td>0.00152</td>
<td>0.00084</td>
</tr>
<tr>
<td>CITI GROUP</td>
<td>0.00194</td>
<td>0.00206</td>
<td>0.00125</td>
<td>0.00081</td>
</tr>
<tr>
<td>GENERAL ELECTRIC</td>
<td>0.00152</td>
<td>0.00125</td>
<td>0.00195</td>
<td>0.00092</td>
</tr>
<tr>
<td>EXXON MOBIL</td>
<td>0.00084</td>
<td>0.00081</td>
<td>0.00092</td>
<td>0.00077</td>
</tr>
</tbody>
</table>

Step 6. Find the risk (standard deviation) for our portfolio

The formula for portfolio dispersion (standard deviation squared) is given by

\[ \sigma_p^2 = \sum \sum \sigma_i \sigma_j \rho_{ij} \]

(taken from http://en.wikipedia.org/wiki/Modern_portfolio_theory)

In matrix form this would look like

\[ \boldsymbol{w}^T \times V \times \boldsymbol{w} \]

In Excel you can write this formula as

\[ =\text{MMULT(}\text{MMULT(} \boldsymbol{w}^T, V \text{)}, \boldsymbol{w}\text{)} \]

Here is the calculation -> 0.00227547

Step 7. Calculate the inverse matrix

Next we create inverse $V$ matrix (or $V^{-1}$ matrix). We use MINVERSE excel function for that.

Highlight the field 4x4 (this is your future inverse matrix). Start entering the formula (the cells remain highlighted): =MINVERSE( Covariation V-matrix 4x4). Then press Ctrl+Shift+Enter (this is important that you should press this combination of button and NOT simply Enter).

Here is what you get:

<table>
<thead>
<tr>
<th></th>
<th>910.94</th>
<th>-112.04</th>
<th>-525.27</th>
<th>-241.54</th>
</tr>
</thead>
<tbody>
<tr>
<td>-112.04</td>
<td>907.61</td>
<td>-230.82</td>
<td>-555.64</td>
<td></td>
</tr>
<tr>
<td>-525.27</td>
<td>-230.82</td>
<td>1579.25</td>
<td>-1071.87</td>
<td></td>
</tr>
<tr>
<td>-241.54</td>
<td>-555.64</td>
<td>-1071.87</td>
<td>3412.62</td>
<td></td>
</tr>
</tbody>
</table>

Step 8. Define 4 scalar values

To define efficient portfolios Huang and Litzenberger determine 4 scalar values: $A$, $B$, $C$ and $D$.

Action 1. $A$ calculation

\[ A = \text{} \text{MMULT(} \text{MMULT(} \text{unity matrix), V^{-1}\text{)}\text{), e-vector} \text{)} \]

First, we multiply matrix $uT$ (unity matrix) and $V^{-1}$ (inverse covariance matrix). We need to highlight four cells and write the formula: =MMULT( V^{-1}-matrix, uT-matrix) Then press Ctrl+Shift+Enter.

Here it is: $A=1.6461$

Action 2. $B$ calculation

\[ B = \text{} \text{MMULT(} \text{eT, V^{-1}\text{)}\text{), e-vector} \text{)} \]

First, we multiply vector $eT$ by matrix $V^{-1}$). Highlight 4 cells and enter the formula: =MMULT( V^{-1}-matrix, eT ) and press Ctrl+Shift+Enter.

Here it is: $B=0.1649$

Action 3. $C$ calculation

\[ C = \text{} \text{MMULT(} \text{unity matrix), V^{-1}\text{)}\text{), e-vector} \text{)} \]

First, we multiply $V^{-1}$ matrix for $uT$ vector (transposed unit vector). Highlight 4 cells and enter the formula: =MMULT( V^{-1}-matrix, uT) and press Ctrl+Shift+Enter.

Here it is: $C=1.6461$

Action 4. $D$ calculation

\[ D = \text{} \text{MMULT(} \text{eT, V^{-1}\text{)}\text{), e-vector} \text{)} \]

First, we multiply vector $eT$ by matrix $V^{-1}$). Highlight 4 cells and enter the formula: =MMULT( V^{-1}-matrix, eT) and press Ctrl+Shift+Enter.

Here it is: $D=0.1649$
Second, multiply the result (uT x V(-1)) by u-vector (unit vector).

Choose a single cell and enter the formula: =MMULT( the result (uT x V(-1)), u-vector) and press Enter

Here is what you get:

\[ \text{C=} \begin{bmatrix} \ 
\end{bmatrix} \begin{bmatrix} 1336.0388 \end{bmatrix} \]

Action 4: D calculation

\[ \text{D=}B \times C-A \times A \]

Choose a single cell and enter the formula with the final values of A, B, C: 

\[ =B \times C-A \times A \]

\[ \text{D=} \begin{bmatrix} 217.5371 \end{bmatrix} \]

**Step 9. Calculation of interim coefficients m and l**

**Action 1. m calculation**

\[ m=V(-1) \times u \]

We multiply V(-1) matrix for u-vector

Highlight 4 cells in a column and enter the formula: =MMULT( V(-1), u ) and press Ctrl+Shift+Enter

\[ \begin{bmatrix} -248.72 \end{bmatrix} \begin{bmatrix} 9.11 \end{bmatrix} \]

**Action 2. l calculation**

\[ l=V(-1) \times e \]

We multiply V(-1) matrix for e-vector

Highlight 4 cells in a column and enter the formula: =MMULT( V(-1), e ) and press Ctrl+Shift+Enter

\[ \begin{bmatrix} -11.271 \end{bmatrix} \begin{bmatrix} -1.067 \end{bmatrix} \]

**Step 10. Calculation of portfolio coordinates**

\[ g = \frac{B \times m - A \times l}{D} \]

\[ h = \frac{C \times l - A \times m}{D} \]

\[ \text{g=} \begin{bmatrix} 5.289 \end{bmatrix} \begin{bmatrix} 79.093 \end{bmatrix} \begin{bmatrix} -11.803 \end{bmatrix} \]

\[ \text{h=} \begin{bmatrix} -1425.126 \end{bmatrix} \begin{bmatrix} 14.989 \end{bmatrix} \begin{bmatrix} -1440.115 \end{bmatrix} \]

**Step 11. Find the effective portfolio for a given return**

Enter an expected return for the portfolio.

Advis: let this number be not really large, because otherwise you'll have to increase leverage significantly

Portfolio return = 1.70%

\[ =114.3% -9.8% -124.8% 120.2% \]

Portfolio risk = 0.23%

**Efficient Portfolio**

\[ \begin{bmatrix} \text{APPLE} & 114.3\% \\ \text{CITIGROUP} & -9.8\% \\ \text{GENERAL ELECTRIC} & -124.8\% \\ \text{EXXON MOBIL} & 120.2\% \end{bmatrix} \]

Portfolio return = 1.70%

Portfolio risk = 0.23%
Step 12. Calculation of efficient portfolio structure with a given amount of money

Enter the amount of money for your portfolio

Money $150,000.00

<table>
<thead>
<tr>
<th>Company</th>
<th>Share in portfolio (%)</th>
<th>Share in portfolio ($)</th>
<th>Last price ($)</th>
<th>Number of shares</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLE</td>
<td>114.3%</td>
<td>171,517</td>
<td>307.47</td>
<td>558</td>
<td>long</td>
</tr>
<tr>
<td>CITI GROUP</td>
<td>-9.8%</td>
<td>-14,635</td>
<td>4.11</td>
<td>-3,561</td>
<td>short</td>
</tr>
<tr>
<td>GENERAL ELECT</td>
<td>-124.8%</td>
<td>-187,202</td>
<td>16.055</td>
<td>-11,660</td>
<td>short</td>
</tr>
<tr>
<td>EXXON MOBIL</td>
<td>120.2%</td>
<td>180,320</td>
<td>66.34</td>
<td>2,718</td>
<td>long</td>
</tr>
</tbody>
</table>

Step 13. Drawing efficient frontier

Drawing of efficient frontier using Huang Litzenberger approach in excel is easy. We just have to make several iterations to find dots on the line.

To set several dots (coordinates) we take our given portfolio return, divide it by 10 and multiply by 1, 2, 3 etc.

In fact you can take any value for portfolio return. We just apply this particular mechanics for automatization of this process.
Here is our efficient frontier:

<table>
<thead>
<tr>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17%</td>
<td>0.0750%</td>
</tr>
<tr>
<td>0.34%</td>
<td>0.0777%</td>
</tr>
<tr>
<td>0.51%</td>
<td>0.0840%</td>
</tr>
<tr>
<td>0.68%</td>
<td>0.0939%</td>
</tr>
<tr>
<td>0.85%</td>
<td>0.1073%</td>
</tr>
<tr>
<td>1.02%</td>
<td>0.1242%</td>
</tr>
<tr>
<td>1.19%</td>
<td>0.1447%</td>
</tr>
<tr>
<td>1.36%</td>
<td>0.1688%</td>
</tr>
<tr>
<td>1.53%</td>
<td>0.1964%</td>
</tr>
<tr>
<td>1.70%</td>
<td>0.2275%</td>
</tr>
</tbody>
</table>

And here are all the results of the model:

**Portfolio return:** 3.00%

**Portfolio risk:** 0.23%

**Amount of money:** 150,000

**Efficient portfolio**

<table>
<thead>
<tr>
<th>Company</th>
<th>Share (%)</th>
<th>Share ($)</th>
<th>Number of shares</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPLE</td>
<td>114.1%</td>
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