MARKOWITS EFFICIENT PORTFOLIO
(HUANG LITZENBERGER APPROACH)

FINANCE TOYS

Huang-Litzenberger approach allows us to find mathematically efficient set of portfolios

Assumptions
There are no limitations on the positions' volumes
You can have both long (buy stocks) and short (sell stocks) positions in your portfolio
We use weekly data, so the expected return is a weekly return and risk is a weekly risk

Step 1. Download stocks' pricing data STEP 1
First, we download pricing data for the stocks
For my model I've chosen four stocks from different sectors (to create a diversified portfolio)
We will construct an effective portfolio on a weekly basis. That means that we will try to find the portfolio with required weekly return and minimal weekly risks (minimal standard deviation)

	APPLE			CITI GROUP			GENERAL ELECTRIC			EXXON MOBIL	
26/03/10	230.9		26/03/10	4.31		26/03/10	18.34		26/03/10	66.54	
02/04/10	235.97	2.2%	02/04/10	4.18	-3.0%	02/04/10	18.33	-0.1%	02/04/10	67.61	1.6%
09/04/10	241.79	2.5%	09/04/10	4.55	8.9%	09/04/10	18.52	1.0%	09/04/10	68.76	1.7%
16/04/10	247.4	2.3%	16/04/10	4.56	0.2%	16/04/10	18.97	2.4%	16/04/10	67.93	-1.2%
23/04/10	270.83	9.5%	23/04/10	4.86	6.6%	23/04/10	19.07	0.5%	23/04/10	69.24	1.9%
30/04/10	261.09	-3.6%	30/04/10	4.37	-10.1%	30/04/10	18.86	-1.1%	30/04/10	67.77	-2.1%
07/05/10	235.86	-9.7%	07/05/10	4	-8.5%	07/05/10	16.88	-10.5%	07/05/10	63.7	-6.0%
14/05/10	253.82	7.6%	14/05/10	3.98	-0.5%	14/05/10	17.64	4.5%	14/05/10	63.6	-0.2%
21/05/10	242.32	-4.5%	21/05/10	3.75	-5.8%	21/05/10	16.42	-6.9%	21/05/10	60.88	-4.3%
28/05/10	256.88	6.0%	28/05/10	3.96	5.6%	28/05/10	16.35	-0.4%	28/05/10	60.46	-0.7%
04/06/10	255.965	-0.4%	04/06/10	3.79	-4.3%	04/06/10	15.71	-3.9%	04/06/10	59.525	-1.5%
11/06/10	253.51	-1.0%	11/06/10	3.88	2.4%	11/06/10	15.56	-1.0%	11/06/10	61.86	3.9%
18/06/10	274.074	8.1%	18/06/10	4.01	3.4%	18/06/10	15.95	2.5%	18/06/10	63.1	2.0%
25/06/10	266.7	-2.7%	25/06/10	3.94	-1.7%	25/06/10	14.91	-6.5%	25/06/10	59.1	-6.3%
02/07/10	246.94	-7.4%	02/07/10	3.79	-3.8%	02/07/10	13.88	-6.9%	02/07/10	56.57	-4.3%
09/07/10	259.62	5.1%	09/07/10	4.04	6.6%	09/07/10	14.95	7.7%	09/07/10	58.78	3.9%
16/07/10	249.9	-3.7%	16/07/10	3.9	-3.5%	16/07/10	14.55	-2.7%	16/07/10	57.96	-1.4%
23/07/10	259.94	4.0%	23/07/10	4.02	3.1%	23/07/10	15.71	8.0%	23/07/10	59.72	3.0%
30/07/10	257.25	-1.0%	30/07/10	4.1	2.0%	30/07/10	16.12	2.6%	30/07/10	59.68	-0.1%
06/08/10	260.091	1.1%	06/08/10	4.06	-1.0%	06/08/10	16.45	2.0%	06/08/10	61.97	3.8%
13/08/10	249.1	-4.2%	13/08/10	3.88	-4.4%	13/08/10	15.38	-6.5%	13/08/10	59.91	-3.3%
20/08/10	249.64	0.2%	20/08/10	3.75	-3.4%	20/08/10	15.03	-2.3%	20/08/10	58.89	-1.7%
27/08/10	241.62	-3.2%	27/08/10	3.76	0.3%	27/08/10	14.71	-2.1%	27/08/10	59.8	1.5%
03/09/10	258.77	7.1%	03/09/10	3.91	4.0%	03/09/10	15.3925	4.6%	03/09/10	61.32	2.5%
10/09/10	263.41	1.8%	10/09/10	3.91	0.0%	10/09/10	15.98	3.8%	10/09/10	61.2	-0.2%
17/09/10	275.37	4.5%	17/09/10	3.95	1.0%	17/09/10	16.29	1.9%	17/09/10	60.78	-0.7%
24/09/10	292.32	6.2%	24/09/10	3.904	-1.2%	24/09/10	16.66	2.3%	24/09/10	61.75	1.6%
01/10/10	282.52	-3.4%	01/10/10	4.09	4.8%	01/10/10	16.36	-1.8%	01/10/10	62.54	1.3%
08/10/10	294.07	4.1%	08/10/10	4.19	2.4%	08/10/10	17.12	4.6%	08/10/10	64.38	2.9%
15/10/10	314.74	7.0%	15/10/10	3.95	-5.7%	15/10/10	16.3	-4.8%	15/10/10	65.19	1.3%
22/10/10	307.47	-2.3%	22/10/10	4.11	4.1%	22/10/10	16.055	-1.5%	22/10/10	66.34	1.8%

Step 2. Find expected return and standard deviation (risk) for each stock STEP 2
Below see the table with basic inputs for the model
Table 1. Basic inputs

	expected return	standard deviation	weights
	e	d	w
APPLE	1.1%	4.9%	114.34%
CITI GROUP	-0.1%	4.6%	-9.76%
GENERAL ELEC	-0.3%	4.5%	-124.80%
EXXON MOBIL	0.0%	2.8%	120.21%

w-column should be empty at first; we will link it to values in Step

e - expected return of the stock. Equal to an average weekly return for a chosen period
d - standard deviation of the stock, which is a measure of risk for the stock. For calculation I use STDEV function (the details of calculation you can find in Excel's help)
w - stock's weight (share) in the portfolio
When constructing the model for the first time, leave the w-column empty (later it will be linked to the formula)

Step 3. Draw two unit-vectors STEP 3
We need them for interim calculations
First: number of columns = 1; number of rows = 4 (same as the number of stocks)
Second: number of columns = 4; number of rows = 1
All the values in vectors equal to 1

Unity vector

u

1
1
1
1

uT

1	1	1	1

Step 4. Draw two transposed matrices for expected returns and weights STEP 4
In table 1 (Step 2) with basic inputs you can see two columns: e (expected return) and w (weights)
Transpose simply means that you should turn columns into rows.
Make links from this matrix to Table 1 values
We add "T" latter in the names of transposed matrices, thus we get wT and eT

Transposed Matrix

	APPLE	CITI GROUP	GENERAL ELECTRIC	EXXON MOBIL
wT	114.34%	-9.76%	-124.80%	120.21%

	APPLE	CITI GROUP	GENERAL ELECTRIC	EXXON MOBIL
eT	1.1%	-0.1%	-0.3%	0.0%

Step 5. Create covariation matrix STEP 5

Covariation defines the dependence of one stock from the other
In covariation matrix we calculate covariation between all stocks
We use COVAR excel function (details on that function and on covariation are available in Excel help)
We call this covariation matrix V

V Covariation matrix

	APPLE	CITI GROUP	GENERAL ELECTRIC	EXXON MOBIL
APPLE	0.00234	0.00119	0.00152	0.00084
CITI GROUP	0.00119	0.00206	0.00125	0.00081
GENERAL ELEC1	0.00152	0.00125	0.00195	0.00092
EXXON MOBIL	0.00084	0.00081	0.00092	0.00077

Step 6. Find the risk (standard deviation) for our portfolio STEP 6

Here is the formula of for portfolio dispersion (standard deviation squared): $\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$

(taken from htt http://en.wikipedia.org/wiki/Modern_portfolio_theory

In matrix form this formula would look like =wT x V x w
In excel you can write this formula as =MMULT(MMULT(wT , V), w)

Here is the calculation -> 0.00227547

Step 7. Calculate the inverse matrix STEP 7

Next we create inverse V matrix (or V (-1) matrix)

We use MINVERSE excel function for that

Highlight the field 4x4 (this is your future inverse matrix)
Start entering the formula (the cells remain highlighted): =MINVERSE(Covariation V-matrix 4x4)
Press Ctrl+Shift+Enter (this is important that you should press this combination of button and NOT simply Enter)

Here is what you get:
V(-1)

910.94	-112.04	-525.27	-241.54
-112.04	907.61	-230.82	-555.64
-525.27	-230.82	1579.25	-1071.87
-241.54	-555.64	-1071.87	3412.62

Step 8. Define 4 scalar values STEP 8

To define efficient portfolios Huang and Litzenberger determine 4 scalar values: A, B, C and D

Action 1. A calculation
A=uT x V(-1) x e

First, we multiply matrix uT (unity matrix) and V(-1) (inverse covariation matrix)
We need to highlight four cells and write the formula: =MMULT(V(-1)-matrix, uT-matrix)
Then press Ctrl+Shift+Enter
You get this:
uT x V(-1)

32.09	9.11	-248.72	1543.57

Second, we multiply the result for e-vector (expected returns)
In a single cell insert the formula: =MMULT(the result from previous calculation (uT x V(-1), e-vector(expected returns))
Then press Enter

Here it is:
A= 1.6461

Action 2. B calculation
B=eT x V(-1) x e

First, we multiply vector eT by matrix V(-1)
Highlight 4 cells and enter the formula: =MMULT(V(-1) , eT) and press Ctrl+Shift+Enter

Here is what you get:
eT x V(-1)

11.60	-1.07	-11.27	2.39

Second, we multiply the result (eT*V(-1) by e-vector (expected return)
Choose a single cell and enter the formula: =MMULT(the result matrix eT x V(-1), e-vector (expected returns)) and press Enter
Here it is:
B= 0.1649

Action 3. C calculation
C=uT x V(-1) x u

First, we multiply V(-1) matrix for uT vector (transposed unit vector)
Highlight 4 cells and enter the formula: =MMULT(V(-1), uT) and press Ctrl+Shift+Enter

Here is the result:
uT x V(-1)

32.09	9.11	-248.72	1543.57

Second, multiply the result (uT x V(-1)) by u-vector (unit vector)
Choose a single cell and enter the formula: =MMULT(the result (uT x V(-1)) , u-vector) and press Enter

Here is what you get:
C= **1336.0388**

Action 4. D calculation
D=B x C-A x A

Choose a single cell and enter the formula with the final values of A, B, C: =B x C-A x A

D **217.5371**

Step 9. Calculation of interim coefficients m and l **STEP 9**

Action 1. m calculation
m=V(-1) x u

We multiply V(-1) matrix for u-vector -------------->
Highlight 4 cells in a column and enter the formula: =MMULT(V(-1), u) and press Ctrl+Shift+Enter

m
32.09
9.11
-248.72
1543.57

Action 2. l calculation
l=V(-1) x e

We multiply V(-1) matrix for e-vector -------------->
Highlight 4 cells in a column and enter the formula: =MMULT(V(-1), e) and press Ctrl+Shift+Enter

l
11.599
-1.067
-11.271
2.385

Step 10. Calculation of portfolio coordinates **STEP 10**

g and h are the two dots of the efficient frontier
g - is the portfolio with minimal expected return
h - is the portfolio with max expected return

g= (B x m - A x l) / D

Here is the calculation
Step by step

1 Calculate B x m	*2* Calculate A x l	*3* B x m - A x l	*4*	g	e	g*e
5.289	19.093	-13.803	APPLE	-6.35%	1.1%	-0.07%
1.501	-1.756	3.257	CITI GROUP	1.50%	-0.1%	0.00%
-41.001	-18.553	-22.448	GENERAL ELI	-10.32%	-0.3%	0.04%
254.457	3.926	250.531	EXXON MOBI	115.17%	0.0%	0.03%

Portfolio return= 0.00%

h = (C x l - A x m) / D

Here is the calculation
Step by step

1 C x l	*2* Am	*3* C x l-A x m	*4*	h	e	h*e
15496.682	52.816	15443.866	APPLE	70.99	1.1%	0.764
-1425.126	14.989	-1440.115	CITI GROUP	-6.62	-0.1%	0.004
-15058.901	-409.409	-14649.492	GENERAL ELECT	-67.34	-0.3%	0.232
3186.568	2540.827	645.741	EXXON MOBIL	2.97	0.0%	0.001

Portfolio return= 100%

Step 11. Find the effective portfolio for a given return **STEP 11**

Enter an expected return for the portfolio
Advise: let this number be not really large, because otherwise you'll have to increase leverage significantly

Portfolio return= **1.7%**

Portfolio return	g	h	h*T	g + hT = w
1.70%	-6.3%	7099.4%	120.7%	114.3%
1.70%	1.5%	-662.0%	-11.3%	-9.8%
1.70%	-10.3%	-6734.3%	-114.5%	-124.8%
1.70%	115.2%	296.8%	5.0%	120.2%
				100.0%

Here is the transposed w matrix. Just make column from the raw

wT			
114.3%	-9.8%	-124.8%	120.2%

Here are the weights of the stocks in the efficient portfolio

VERY IMPORTANT!
Make links from column g + h x T = w to the column w (weights) in table 1, Step 2

Portfolio risk calculation
to find portfolio risk we should multiply three matrices: V, w and wT
Choose a cell and enter the formula: =MMULT(MMULT(wT, V), w) and press enter

Portfolio risk= **0.23%**

EFFICIENT PORTFOLIO	
APPLE	**114.3%**
CITI GROUP	**-9.8%**
GENERAL ELECTRIC	**-124.8%**
EXXON MOBIL	**120.2%**

Portfolio return= **1.70%**
Portfolio risk= **0.23%**

Step 12. Calculation of efficient portfolio structure with a given amount of money

STEP 12

Enter the amount of money for your portfolio

Money= **150,000.00** $

Company	Share in portfolio (%)	Share in portfolio ($)	Last price ($)	Number of shares	Position
APPLE	114.3%	171,517	307.47	558	long
CITI GROUP	-9.8%	-14,635	4.11	-3,561	short
GENERAL ELEC`	-124.8%	-187,202	16.055	-11,660	short
EXXON MOBIL	120.2%	180,320	66.34	2,718	long

Step 13. Drawing efficient frontier

STEP 13

Drawing of efficient frontier using Huang Litzenberger approach in excel is easy. We just have to make several iterations to find dots on the line

To set several dots (coordinates) we take our given portfolio return, divide it by 10 and multiply by 1, 2, 3 etc
In fact you can take any value for portfolio return. We just apply this particular mechanics for automatization of this process

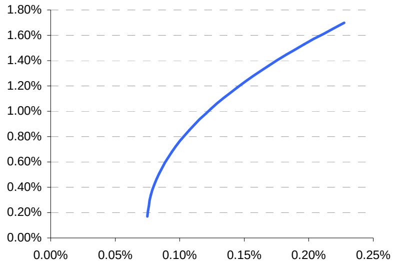
Dots	1	2	3	4	5	6	7	8	9	10
Expected portfolio return	0.2%	0.3%	0.5%	0.7%	0.9%	1.0%	1.2%	1.4%	1.5%	1.7%

Then we do calculations from the Step 11 for all the dots (all the portfolio returns)

Expected return T	g	h	h*T	g + hT	wT				Risk
0.17%	-6.3%	7099.4%	12.1%	5.7%	5.7%	0.4%	-21.8%	115.7%	0.0750%
0.17%	1.5%	-662.0%	-1.1%	0.4%					
0.17%	-10.3%	-6734.3%	-11.4%	-21.8%					
0.17%	115.2%	296.8%	0.5%	115.7%					
				100.0%					
	g	h	h*T	g + hT	wT				
0.34%	-6.3%	7099.4%	24.1%	17.8%	17.8%	-0.8%	-33.2%	116.2%	0.0777%
0.34%	1.5%	-662.0%	-2.3%	-0.8%					
0.34%	-10.3%	-6734.3%	-22.9%	-33.2%					
0.34%	115.2%	296.8%	1.0%	116.2%					
				100.0%					
	g	h	h*T	g + hT	wT				
0.51%	-6.3%	7099.4%	36.2%	29.9%	29.9%	-1.9%	-44.7%	116.7%	0.0840%
0.51%	1.5%	-662.0%	-3.4%	-1.9%					
0.51%	-10.3%	-6734.3%	-34.3%	-44.7%					
0.51%	115.2%	296.8%	1.5%	116.7%					
				100.0%					
	g	h	h*T	g + hT	wT				
0.68%	-6.3%	7099.4%	48.3%	41.9%	41.9%	-3.0%	-56.1%	117.2%	0.0939%
0.68%	1.5%	-662.0%	-4.5%	-3.0%					
0.68%	-10.3%	-6734.3%	-45.8%	-56.1%					
0.68%	115.2%	296.8%	2.0%	117.2%					
				100.0%					
	g	h	h*T	g + hT	wT				
0.85%	-6.3%	7099.4%	60.3%	54.0%	54.0%	-4.1%	-67.6%	117.7%	0.1073%
0.85%	1.5%	-662.0%	-5.6%	-4.1%					
0.85%	-10.3%	-6734.3%	-57.2%	-67.6%					
0.85%	115.2%	296.8%	2.5%	117.7%					
				100.0%					
	g	h	h*T	g + hT	wT				
1.020%	-6.3%	7099.4%	72.4%	66.1%	66.1%	-5.3%	-79.0%	118.2%	0.1242%
1.020%	1.5%	-662.0%	-6.8%	-5.3%					
1.020%	-10.3%	-6734.3%	-68.7%	-79.0%					
1.020%	115.2%	296.8%	3.0%	118.2%					
				100.0%					
	g	h	h*T	g + hT	wT				
1.19%	-6.3%	7099.4%	84.5%	78.1%	78.1%	-6.4%	-90.5%	118.7%	0.1447%
1.19%	1.5%	-662.0%	-7.9%	-6.4%					
1.19%	-10.3%	-6734.3%	-80.1%	-90.5%					
1.19%	115.2%	296.8%	3.5%	118.7%					
				100.0%					
	g	h	h*T	g + hT	wT				
1.36%	-6.3%	7099.4%	96.6%	90.2%	90.2%	-7.5%	-101.9%	119.2%	0.1688%
1.36%	1.5%	-662.0%	-9.0%	-7.5%					
1.36%	-10.3%	-6734.3%	-91.6%	-101.9%					
1.36%	115.2%	296.8%	4.0%	119.2%					
				100.0%					
	g	h	h*T	g + hT	wT				
1.53%	-6.3%	7099.4%	108.6%	102.3%	102.3%	-8.6%	-113.4%	119.7%	0.1964%
1.53%	1.5%	-662.0%	-10.1%	-8.6%					
1.53%	-10.3%	-6734.3%	-103.0%	-113.4%					
1.53%	115.2%	296.8%	4.5%	119.7%					
				100.0%					
	g	h	h*T	g + hT	wT				
1.70%	-6.3%	7099.4%	120.7%	114.3%	114.3%	-9.8%	-124.8%	120.2%	0.2275%
1.70%	1.5%	-662.0%	-11.3%	-9.8%					
1.70%	-10.3%	-6734.3%	-114.5%	-124.8%					
1.70%	115.2%	296.8%	5.0%	120.2%					
				100.0%					

Here is our efficient frontier

	x-axis Return	y-axis Risk
1	0.17%	0.0750%
2	0.34%	0.0777%
3	0.51%	0.0840%
4	0.68%	0.0939%
5	0.85%	0.1073%
6	1.02%	0.1242%
7	1.19%	0.1447%
8	1.36%	0.1688%
9	1.53%	0.1964%
10	1.70%	0.2275%



And here are all the results of the model

Portfolio return= 3.00%
Portfolio risk= 0.23%

Amount of money= 150,000

Efficient portfolio

Company	Share (%)	Share ($)	Number of shares	Position
APPLE	114.3%	171,517	558	long
CITI GROUP	-9.8%	-14,635	-3,561	short
GENERAL ELECTI	-124.8%	-187,202	-11,660	short
EXXON MOBIL	120.2%	180,320	2,718	long